Varianta 62

SUBIECTUL I

a) $AB = 2\sqrt{2}$.
b) M este mijlocul segmentului (AC) $\Rightarrow M(2,1)$.
c) $MB = 3$.
d) $S_{ABC} = 6$.
e) $\cos^2 30^\circ = \frac{3}{4}$.
f) $\bar{z} = -3 - 6i$

SUBIECTUL II

1.
a) $1^5 - 2^4 + 3^3 - 4^2 + 5^1 = 1$.
b) $(f \circ f)(2) = 16$.
c) $p = \frac{3}{4}$.
d) $C_{20}^2 = 190$.
e) $x = 3$.

2.
a) $f'(x) = 5x^4$.
b) $\lim\limits_{x\to 1}\frac{f(x)-f(1)}{x-1} = 5$.
c) $f'(x) \geq 0,\ \forall x \in \mathbf{R} \Rightarrow$ funcția f este crescătoare pe **R**.
d) $\int_0^1 f(x)dx = \frac{1}{6}$.
e) $\lim\limits_{x\to\infty}\frac{f(x)}{x \cdot f'(x)} = \frac{1}{5}$.

SUBIECTUL III

a) $A^2 = I_3$.
b) Din **a)** $\Rightarrow A^2 = I_3 \Rightarrow A^{2007} = (A^2)^{1003} \cdot A = (I_3)^{1003} \cdot A = I_3 \cdot A = A$ **.**

c) $A \circ A = A + A - 3I_3 = \begin{pmatrix} -3 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & -3 \end{pmatrix}$.

d) $X \circ Y = X + Y - 3I_3$, $Y \circ X = Y + X - 3I_3 \Rightarrow X \circ Y = Y \circ X$, $\forall X, Y \in M_3(\mathbf{R})$.

e) $(X \circ Y) \circ Z = X + Y + Z - 6I_3 = X \circ (Y \circ Z)$

f) $X \circ E = X \Leftrightarrow X + E - 3I_3 = X \Leftrightarrow E = 3I_3$.

g) Fie $P(n): \underbrace{X \circ X \circ ... \circ X}_{de\, n\, ori} = nX - 3(n-1)I_3$, n$\in$**N*.**

$P(1): X = X$, adevărată.. Presupunem $P(k)$ (A) şi demonstrăm $P(k+1)$ (A), k≥1.

$P(k): \underbrace{X \circ X \circ ... \circ X}_{de\, k\, ori} = kX - 3(k-1)I_3$, $P(k+1): \underbrace{X \circ X \circ ... \circ X}_{de\, k+1\, ori} = (k+1)X - 3kI_3$.

Dar $\underbrace{X \circ X \circ ... \circ X}_{de\, k+1\, ori} = \left(\underbrace{X \circ X \circ ... \circ X}_{de\, k\, ori}\right) \circ X \overset{P(k)}{=} (kX - 3(k-1)I_3) \circ X = (k+1)X - 3kI_3$.

De unde $P(n)$ este (A), ∀n$\in$**N*.**

SUBIECTUL IV

a) $f_0(0) = 0$.

b) $f_1(x) = f_0'(x) = e^x(x+1)$.

c) $\int_0^1 f_1(x)dx = \int_0^1 (f_0(x))' dx = e$.

d) Fie $P(n): f_n(x) = e^x(x+n)$, n$\in$**N.**

$P(0): f_0(x) = e^x x$, (A).

Presupunem $P(k)$ (A) si demonstrăm $P(k+1)$ (A), k≥0.

$P(k): f_k(x) = e^x(x+k)$, iar $P(k+1): f_{k+1}(x) = e^x(x+k+1)$.

Dar $f_{k+1}(x) = f_k'(x) = \left(e^x(x+k)\right)' = e^x(x+k) + e^x = e^x(x+k+1)$ (A).

De unde $P(n)$ este (A) ∀n$\in$**N*.**

e) $a_n = f_1(0) + f_2(0) + ... + f_n(0) = 1 + 2 + ... + n = \frac{n(n+1)}{2}$.

f) $\lim_{n\to\infty} \frac{a_n}{n^2} = \lim_{n\to\infty} \frac{n(n+1)}{2n^2} = \frac{1}{2}$.

g) Vom demonstra că $f_{n+1}(x) \geq f_n(x)$, $\forall n \in$ **N,** $\forall x \in [0,1]$.

$f_{n+1}(x) \geq f_n(x) \overset{\mathbf{d)}}{\Leftrightarrow} e^x(x+n+1) \geq e^x(x+n) \Leftrightarrow x+n+1 \geq x+n \Leftrightarrow 1 \geq 0$ (A).

Aplicând monotonia integralei definite $\Rightarrow \int_0^1 f_{n+1}(x)dx \geq \int_0^1 f_n(x)dx$.